A
TREATISE
ON THE
HORIZONTAL SUN AND MOON,

Wherein is ſhewn, according to the Principles of Refraction,
HOW IT HAPPENS, THAT THOSE BODIES SEEM BIGGER IN THE HORIZON THAN IN THE ZENITH;

AND ALSO

HOW MANKIND DO NOT FORM THEIR JUDGMENT CONCERNING THE MAGNITUDE OF OBJECTS BY THE ANGLES, WHICH THEY SUBTEND AT THE EYE, EITHER WHEN THEY ARE SEEN IN REFRACTED OR UNREFRACTED LIGHT;

AND LIKEWISE

HOW THE SUPERIOR PLANETS MAY HAVE A RETROGRADE MOTION IN OPPOSITION, ALTHO' THE SUN MOVES IN AN ORBIT ROUND THE EARTH;

TOGETHER
WITH CERTAIN ASTRONOMICAL DIFFICULTIES.

BY THE REV. JAMES ATKINS,
RECTOR OF ST. MICHAEL'S IN LONG-STANTON IN CAMBRIDGESHIRE, AND FORMERLY FELLOW OF ST. MARY MAGDALENE COLLEGE IN CAMBRIDGE.

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TO THE READER.

I RECKON it needful to acknowledge, that I am, for my part, convinced by the experiments and reaſonings, which I have put together in the following treatiſe; and that, on the ſtrength of ſuch conviction I have riſked the hazard of laying them thus before the world. If however people, who are ſkilful in optical matters, ſhou'd diſcover any falſe ſtep in the undertaking, which I make known after this manner, I ſhou'd look on myſelf to be much indebted to them, if they wou'd be ſo far favourable, as to acquaint me either publickly or privately with an account of it. For I wou'd be underſtood to teach, what I do teach, on this occaſion, rather with a hope of receiving inſtruction than of giving it.

JAMES ATKINS.

ON THE

HORIZONTAL SUN AND MOON.

PROPOSITION I.

A PROBLEM. (fig. 1.)

WHEN diverging rays fall on a denſer and ſpherical refracting medium, it is required to find how the foci, to which they will converge after refraction, will be placed with reſpect to each other at different degrees of obliquity; and alſo how the laſt images, from which they will ſeem to come at different degrees of obliquity, are placed, with reſpect to each other, if they ſtill continue to diverge after refraction.

Let *LMN* repreſent the ſpherical ſurface of a refracting medium, which is more denſe than the medium, by which it is ſurrounded. And let the point *R* be a radiant or focus, from which come a pencil of rays, that fall on it. And let *RMF* be the axis of ſuch pencil. And let the refracting power of the medium be ſuch, that the ray *RX* ſhall be made to converge after refraction towards ſome point *F*, then the ray *RZ*, which falls on the refracting ſurface *LMN* further from the axis of the pencil *RMF* than the ray *RX*, will converge to ſome point *f* nearer to the refracting ſurface *LMN* than the point *F*, and if it be in the ſame plane as the ray *RXF*, will cut or meet it in ſome point *a**. Now if the refracting power of the medium ſhould decreaſe, the points *F* and *f* would remove further

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* See Smith's Opticks, Vol. I. Art. 69. fig. 131.

further from the refracting ſurface LMN, but the point F will be furtheſt from it. Let then the refracting power of this medium ſo far decreaſe, that the ray XF after refraction ſhall be made to become parallel to the axis of the pencil RMF (ſee fig. 2.) and that the point F ſhall be removed to an infinite diſtance; then the ray Zf, which was endued with a greater degree of convergency, will ſtill converge to ſome point f in the axis RMF at a finite diſtance, and will cut or meet the ray XF as before in ſome point a. Let then the refracting power of this medium ſo much further decreaſe, that the ray Zf may become parallel to the axis of the pencil RM (ſee fig. 3.) then the ray RX after refraction will be made to diverge from ſuch axis in the direction FXD, and will ſeem to come from ſome point F further from the refracting ſurface than the point R. But altho' the refracted rays Zf and XD are one of them parallel to the axis RM, and the other diverging from it, yet they will be converging, with reſpect to each other. In like manner, if the refracting power of the medium ſhould decreaſe ſomewhat further, the ray Zf, as well as the ray XD, will alſo diverge from the axis of the pencil RM. Yet the decreaſe in ſuch refracting power may be ſo ſmall, and the difference of obliquity between the points X and Z may be ſo great, that the rays Zf and XD will both of them diverge from the axis of the pencil RM, but ſtill, as before, converge with reſpect to each other. But altho' the change in the refracting power can cauſe ſuch a convergency, yet the point Z can approach ſo near to the point X as to prevent it, and to render the rays Zf and XD diverging not only from the axis of the pencil RM, but alſo from each other, (ſee fig. 4.) Since then the ray zf (fig. 5.) can be diverging from the ray XD, and the ray Zf can be at the ſame time converging to it, there muſt be ſome point O between the points Z and z, where the refracted ray Oq will be parallel to the refracted ray XD. Now if the rays RM and RX are ſo near to each other as to enter the eye at the ſame time, they will ſeem to come after refraction from ſome point F, which is common to both of them. And if the rays RX and Rz (fig. 6.) are ſo near to each other as to enter the eye at the ſame time, they will ſeem to come after refraction from ſome point g, which is common to the produced lines XF and zf. Since then the point g is below the axis RM, and further from the refracting ſurface LMN than the point F, any other rays, which are more oblique than the ray Rz, and leſs oblique than the ray RO, and which enter the eye at the ſame time, will ſeem to come from points, which are further below the axis RM, and further from the ſurface LMN, the more near are the points to the point O, where ſuch rays fall on the ſurface. But if the rays RX and RO are ſo near to each other as to enter the eye at the

ſame

ſame time, they will, being parallel, ſeem to come after refraction from an infinite diſtance. And if the rays *RX* and *RZ* are ſo near to each other as to enter the eye at the ſame time, they will, being converging after refraction, ſeem to come from a diſtance, which is greater than an infinite diſtance.

Cor. Hence, altho' the point *R*, when ſeen by refraction at ſome degree of obliquity from the axis of the pencil *RM*, will always ſeem raiſed above that axis; Yet in all inſtances, except when the rays after refraction become parallel to it or converging to it, they will ſeem to come from a laſt image, which is in ſome point *g* below it.

PROPOSITION II.

If a man be placed within a denſer and ſpherical refracting medium; and if there be an object ſituated at a great diſtance from it, what optical writers call the laſt image of ſuch an object, will be further off and greater, the more obliquely the rays by which he ſees it fall on the ſurface of the medium; but whatever be their obliquity, it will, under all its changes, be of ſuch a ſize and at ſuch a diſtance as to ſubtend the ſame angle.(fig. 6.)

Let *LMN* repreſent the ſurface of a denſer and ſpherical refacting medium, whoſe centre is *C*; and *ywv* repreſent an inner ſpherical ſurface, which is concentric with it; and let *ABD* be an object placed at ſuch a diſtance from the outer ſurface of the medium *LMN*, that the ſemidiamiter *CM* ſhall bear no ſenſible proportion to it; and from the centre *C*, let the produced radius *CMB* fall perpendicularly on the object *ABD*. Now if the rays *Bx* and *Bz* and all the other rays, which come from the point *B*, where the produced raidus *CMB* meets the object *ABD*, fall on the ſurface *LMN*, with different degrees of obliquity, or at different diſtances from the produced ſemidiameter *CMB*, they will after refraction become either converging or parallel, or leſs diverging, and ſeem to come from laſt images, as we have explained in the foregoing propoſition. Thus thoſe rays, which are rendered converging, will, according to their convergency, ſeem to come from a diſtance, which is (more or leſs) greater than an infinite diſtance. Thus likewiſe thoſe rays, which are rendered parallel, will ſeem to come only from an infinite diſtance. And thoſe rays, which are rendered leſs diverging, will according, to their divergency, ſeem to come from different finite diſtances. As the ray *Bz*, which is rendered leſs diverging, will ſeem to come from ſome point *p*, which is further off from the ſurface *LMN*, than the point *b*, from which the ray *Bx*, whoſe divergency is leſs decreaſed, will ſeem to come. In like manner, the rays coming from the point *A*, and falling on the refracting ſurface *LMN* about the points Z and *X*, will ſeem to come after refraction from the points *o* and *a*; and the rays coming from the point *D*, and falling about the ſame points Z and *X* will ſeem to come after refraction from the points *q* and *d*. But ſince the produced radii *CAao*, and *CDdq* bound the laſt images *abd* and *opq*, they will at the centre *C* ſubtend the ſame angle; and ſince the ſemidiameter *CM* bears no ſenſible proportion to the diſtances *bM* and *pM*, the laſt image *abd* will ſubtend the ſame angle at the point *w*, as the laſt image *opq* will ſubtend at the point *y*.

PROPOSITION III.

The laſt images of the ſun and moon are much greater when ſeen in the horizon than in the zenith. (fig. 6.)

Since the atmoſphere is a ſpherical refracting medium, more denſe than the medium by which it is ſurrounded, and ſince the earth's ſemidiamter bears but an inconſiderable proportion to the diſtance of the heavenly bodies; every thing will be true with reſpect to it, which has been ſhewn or proved to be true in the two foregoing propoſitions. Let then *LMN* repreſent the ſurface of the atmoſphere, and let the produced radius *CMB* reach from the centre of the atmoſphere to the centre of the moon; and let *Bx* be a ray of light, which falls on the atmoſphere at the diſtance of one degree from ſuch produced radius *CMB*. Now becauſe the ſemidiameter of the earth, when ſeen at the moon will ſubtend an angle of one degree, and becauſe the arc *LM* at the end *M* is leſs inclined to the moon than at the end *L*, the angle *xBM* will not be leſs than one ninetieth part of a degree or forty-five ſeconds. Whence the angle *xBb* will be an hundred and ſeventy-nine degrees and fifteen ſeconds, and the angle *Bxb*, according to Sir Iſaac Newton's table of refractions, (Philoſoph. Tranſact. N°. 368.) will be one ſecond, and the remaining angle *Bbx* will be forty-four ſeconds. But the ſides of triangles are to each other as the ſines of the oppoſite angles, (ſee Smith's Opticks, 4 Chapters) and the ſines of ſmall angles are to each other, or very nearly to each other as the angles, which they ſubtend. Whence the ſide *Bx* of the triangle Bxb is to the ſide Bb in the proportion of forty-four to one. Hitherto I have taken for granted, that the arc xm ſubtends at the moon an angle of only forty-five ſeconds; but becauſe the whole arc LM ſubtends an angle of one degree, and becauſe at the end L it is much more bent from the moon than at the end M, the arc MX muſt ſubtend at the moon an angle greater than forty-five ſeconds, and the ſide Bx of the triangle Bxb muſt be to the ſide Bb, in no leſs a proportion than of ſixty or ſeventy to unity, and the laſt image of the moon, when ſeen at the diſtance of one degree from the zenith, muſt be behind the moon not more than a ſixtieth or ſeventieth part of the moon's diſtance from the earth. Now let the ray of light BZ fall on the atmoſphere at the diſtance of ninety degrees from the ray of light or produced radius CMB, then the angle ZBM will be one degree, and the angle ZBp will be an hundred and ſeventy-nine degrees; and the angle BZp, according to Sir Iſaac Newton's table of refractions, is thirty-three minutes and forty-five ſeconds. Whence the angle BpZ is twenty-ſix minutes and fifteens ſeconds; and the ſide Bp of the angle BZp, which ſubtends the greater angler BZp, muſt be greater than the ſide BZ, which ſubtends the leſs angle BpZ. But altho', according to the firſt propoſition, the point b might be the laſt image of the point B in the former inſtance, if the ray Bx and the axis BM entered the eye together; yet according to the ſame propoſition, the point p in this inſtance cannot in like manner be the laſt image of it. For if the ray BZ and the other rays, which come from the point B and enter the eye along with it, are rendered parallel by refraction, the laſt image of the point B when ſeen at the point Z will be at an infinite diſtance; and if they are rendered converging by refraction, it will be at a diſtance greater than an infinite diſtance; and if they are rendered leſs diverging, it will be below the axis of the pencil Bbp, and perhaps as far beyond the point of interſection p, as that point is diſtant from the earth, and ſince the line Bp is greater than the line BZ, that point will be more than twice the moon's diſtance from it. Becauſe then the laſt image of the point B, or of the whole moon, is ſo much further diſtant when ſeen in the horizon than in the zenith; ſuch laſt image muſt likewiſe be proportionally greater in one ſituation than in the other.

If the earth, as is commonly held, ſubtends no angle at the ſun, all the rays coming from the latter, and falling on the atmoſphere, will be more or leſs converging after refraction; and ſince they will ſeem to come from a diſtance, which is further beyond an infinite diſtance, when the ſun is ſeen in horizon than when ſeen in the zenith, the ſun's laſt image will be greater, when beheld in one point of view than in the other.

Cor. Since the laſt images of the ſun and moon, and other heavenly bodies, are ſo much further beyond thoſe bodies in the horizon than in the zenith, it is ſubmitted, whether the reſpective diſtances of ſuch laſt images are not the cauſes, why a clear ſky does not appear to be a hemiſphere, or ſomewhat greater than a hemiſphere, rather than a ſegment of a ſphere leſs than a hemiſphere.

PROPOSITION IV.

If a ſtraight line ſhould repreſent an object, and if the eye of a ſpectator ſhould approach towards it along another ſtraight line, by which it is biſected, and to which it is perpendicular, the angle ſubtended at the eye by ſuch ſtraight line or object will increaſe ſomewhat faſter, than the diſtance of the eye from the top of it decreaſes. (fig. 7.)

Dem. About the centre *C* with any diſtance *CA* deſcribe the arc *ADB*, and join the points *A* and *B* by the line *AB* and biſect the line *AB* in the point *O*, and join the points *C* and *O* by the line *CO*, and produce the line *CO* at the end *O*, and from the produced line *CO* beyond the point *D*, where it meets the arc *ADB*, cut off the line *Db* equal to the line *CE*, which is any part of the line *CO* at the other end of it; and about the centre *E* with the diſtance *Eb* deſcribe the arc *eabdf*, and join the points *E* and *A* by the line *EA*, and the points *E* and *B* by the line *EB*, and produce the lines *EA* and *EB* till they meet the arc *eabdf* in the points *f* and *e*, and join the points *C* and *A*, *C* and *B*, by the lines *CA* and *CB*, and let thoſe lines be produced, till they meet the arc *eabdf* in the points *d* and *a*; and ſince the line *CD* is equal to the line *Eb*, the arcs *ADB* and *eabdf* will be of the ſame curvature and the angle *bEf* will be to the angle *DCA*, as the arc *bdf* is to the arc *DA*. Now let the points *e* and *f*, where the line *EBe* and *EAf* meet the arc *eabdf*, be joined by the line *emf*, and the triangle *mEf* will be ſimilar to the triangle *OEA*, and converſely the line *mf* will be to the line *OA* as the line *fE* or the line *AC*, which is equal to it, is to the line *AE*. But the arc *bf* bears a greater proportion to the line *mf*, which is the ſine of the angle *OEA*, than the arc *DA* bears to the line *OA*, which is the ſine of the angle *OCA*. Whence the angles *OEA* and *OCA* bear a greater proportion to each other than the lines or ſines *mf* and *OA*, or than the lines *AC* and *AE*, which are pro-

proportional to them.—If then the line *AOB* ſhould repreſent an object; and if the eye of a ſpectator ſhould move towards it from the point *C* along the line *CO*, the angle ſubtended by it at any place, from which it is ſeen, will increaſe ſomewhat faſter than the diſtance of the eye from the top of it decreaſes.

Cor. Hence, if any body ſhould look at a ſteeple or building at a diſtance of two hundred yards from the top of it, the height of ſuch ſteeple or building would ſubtend at his eye an angle, leſs than half the angle, which would be ſubtended at his eye by it, if he ſtood but at the diſtance of an hundred yards from the top of it.

PROPOSITION V.

Men do not form their judgements concerning the apparent magnitude of objects by the angle, which they ſubtend at the eye.

The height of any object at which we look, according to the corollary of the laſt propoſition, will ſubtend an angle leſs than half the angle which would be ſubtended by it, if we ſtood from the top of it but at half the diſtance of the place, at which we now ſtand to look at it. For inſtance, the height of a ſteeple or other building, when ſeen at two hundred yards diſtance from the top of them, will ſubtend an angle leſs than half the angle, which they would reſpectively ſubtend, if ſeen at an hundred yards diſtance from the top of them. Yet if we ſhould approach towards ſuch ſteeple or, building or withdraw from them, their apparent magnitude would by no means increaſe, during ſuch approach, or leſſen, during ſuch retreat, in proportion to the increaſe or decreaſe of the angles, thus ſubtended by them. It is allowed indeed, that a ſteeple or building will appear leſs at ten miles diſtance, than at an hundred yards diſtance, but not ſo much ſo, as the angle ſubtended by them at the former diſtance, will be leſs than the angle ſubtended by them at the latter diſtance. For theſe reaſons then, whatever be the principle or rule, by which we are guided in our judgement concerning the apparent magnitude of objects, it will not keep pace with the angles which they ſubtend at the eye.*

Cor. Hence, ſince any two equal objects as two ſteeples, when ſeen, one at two hundred yards diſtance from the top of it, and the other at one hundred yards diſtance from the top it, will appear equal or nearly equal to each other, although they ſubtend very different angles at the eye, if the former ſhould be ſo increaſed

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* For further information on this ſubject, ſee the following page.

creaſed, that both of them may ſubtend the ſame angle, it will then appear much the greateſt. For it appeared already equal, or nearly equal to the latter, and it muſt appear much greater, when it is ſo increaſed, as to become, according to the firſt propoſition, more than twice as great in its height and breadth, as what it was before.

ON THE MEASURE, BY WHICH WE FORM OUR JUDGEMENT CONCERNING THE MAGNITUDE OF DISTANT OBJECTS.

Since we never form our judgement concerning the apparent magnitude of near objects by the angles, which are ſubtended by them at the eye, we may very well doubt, whether we form our judgement concerning the apparent magnitude of diſtant objects by angles of the ſame ſort. For altho' ſome people believe, that, with reſpect to the former kind of objects, we correct the judgement of our eye-ſight by experience, yet it is not very eaſy to ſhew, as it ſeems to me, how it happens, that we are no leſs furniſhed with this experience in our earlieſt youth, than when we arrive at years of maturity. For inſtance, if a boy of three years old, thinks, and always remembers to have thought, his father or his mother, or a cow or a horſe, or any other object, with which he is acquainted, to be of the ſame ſize, whether he ſees them at five yards diſtance, or at an hundred yards diſtance, we ſhould like to underſtand, from what footſteps of his life we can gather, that he corrects the judgement of his eye-ſight by uſe, or knowledge, or practice. Becauſe if objects, according to his remembrance, have always appeared to him, as they now appear to him, they afford ſuch a proof, that he is not indebted to any acquired ſkill in the judgement, which he forms concerning their apparent magnitude, as nothing can overturn but a clear and poſitive proof, that they have appeared very differently to him. Yet Mr. Locke tells us, that ſpherical objects will raiſe in young children or, in people, who have been blind, and have lately gained their eye-ſight, the idea of flatneſs, or of a circular plane, as if objects of all ſhapes and at all diſtances would appear to them in the ſame plane, or at the ſame diſtance, and as if they were in a picture, and would ſeem of ſuch a ſize as agrees with the meaſured or geometrical angle, which they ſubtend at the eye. But does Mr. Locke mean, that a ſpherical object, at a near diſtance, will raiſe the ſame idea as a circular plane, which has an equal diameter, and is placed at an equal diſtance perpendicularly to a line drawn from the eye to itſelf? Or if ſuch is his meaning, have we not good room to aſk, by what fact he could have eſtabliſhed the truth or reaſonble-

who

neſs of it? Becauſe, for my part, I have never heard of any body, who can recollect to have ſeen objects after that manner; neither do I know of any reaſon, why we ſhou'd think that young children and grown perſons ſee much unlike each other. It is true indeed, that objects appear ſomewhat leſs to a man than to a child, and can be ſeen at greater diſtances by one than by the other; yet when they are within the reach of a child's eye, and are ſeen by it at different diſtances they will undergo the ſame changes in the appearance of their ſize, as if a man looked at them. For take a chicken, which has been newly hatched, and lay ſome crumbs of bread before it, and you will find, that it will ſee them and peck at them with the ſame certainty with which an old hen would ſee them and peck at them; or take a duckling, as ſoon almoſt as it is come from the egg, and put it into the water, and you will ſee, that it will immediately be aware of the flies, and will ſtraightway ſwim towards them and catch them. Now altho' a young child is more helpleſs than a chicken or duckling, yet its ſenſe of ſeeing, like the ſenſe of ſeeing in a chicken or a duckling, and like all its own other ſenſes of hearing and ſmelling and taſting and ſeeing is, as I venture to hold, rather the gift of heaven than the fruit of experience. But if it ſhould be ſaid, on the other ſide, that the eyes of children learn to ſee by the ſame ſteps and in the ſame way as the eyes of couched people, it may be anſwered, that all objects alike appear to couched eyes very large and ſo near, that they touch or ſeem touch them. (See Smith's Opticks, Vol. I. page, 73.) Whence there is either no reſemblance between the eyes of children and couched eyes; or if there is any reſemblance, objects muſt appear to the former as large and as near as to the latter, and neither at firſt nor afterwards of ſuch a ſize, as exactly agrees with the real or meaſured or geómetrical angle, which they ſubtend at the eye. Whether then the power, which we enjoy, of meaſuring the apparent magnitude of objects by our eye-ſight was given us at our birth, as we now have it, like the limbs of our body; or whether it has been learned by us like other arts, yet we are never wholly guided in it by the real angles, which ſuch objects ſubtend at the place, from which we take our view of them. For if the eye learns this power of judging like couched eyes by experience; like couched eyes alſo it will not meaſure the apparent magnitude of objects by ſuch angles; neither will our eyes meaſure the apparent magnitude of objects by ſuch angles, if this power of judging is not learned by experience, and if our eyes are at firſt ſkilful like chickens eyes. For then they will both in childhood and at our full age, meaſure by the ſame rules the apparent magnitude of all objects near and diſtant, and if an object is perpendicular to the plane of a ſpectors eye and approaches towards it, its apparent mag-

nitude

nitude will increaſe leſs faſt, than the angle which it ſubtends, increaſes; and if it withdraws from the eye, its apparent magnitude will decreaſe leſs faſt than the angle which it ſubtends, decreaſes. See Prop. 5. Cor.

The opinion, which men entertain, that the eye learns to correct the judgement of its eye-ſight by experience, ſeems to have ariſen from what are called the deceptions of ſight, to which we are liable. Yet ſuch deceptions of ſight are not owing to the eye itſelf, unleſs it be unſound, but to the changes and refractions, which the rays of light undergo, before they fall on it. Thus take a ſtraight ſtick of the ſame thickneſs through its whole length, and dip it halfway into the water, and it will forthwith appear bent in the middle, and thicker at one end than at the other. Now it is well known, that it is not indebted for this change in its appearance to any change in the eye, but to the refraction of the water, and that the eye cannot by any effort in its power correct the judgment of its ſight concerning it. For it will ſeem bent in the middle, and thicker at one end than at the other in ſpite of every endeavour, which the eye can make to ſee it otherwiſe. It is ſaid likewiſe, that if we look at men on the earth from a high building, that they will ſeem leſs, than if we were on the earth along with them; and that if we ſtand in a valley between two high hills, and ſee crows fly acroſs it, they will ſeem higher than we ſhall find that they are, when they approach to either ſide of it. But to make either of theſe inſtances, even if they be true, of any uſe in our preſent argument, it will be needful to ſhew, what has not yet been ſhewn, that the men and crows, of which they make mention, appear of ſuch a ſize, at different diſtances, as exactly agrees with the real or meaſured or geometrical angles, which they ſubtend at the eye; and alſo, that we can learn to ſee them by practice in theſe ſituations as they are wont to appear in all other ſituations.

It is very hard to illuſtrate by any example or reſemblance, that the angles ſubtended at the eye by diſtant objects are not the guides, by which it forms its judgement concerning their apparent magnitude. For ſince we can only reaſon from thoſe objects, which are near to us, it may almoſt always be ſaid, that we are already acquainted with their ſize and are under the influence of prepoſſeſſions; or that we are aware of the different diſtances, at which they are from us, and cannot avoid making allowances for them. Yet if we place ourſelves behind a low wall and peep over it, we may look at the tops of trees beyond it without ſeeing their intermediate diſtances, and I can witneſs for myſelf, that great trees, when viewed after this manner, will appear greater than leſs trees, altho' the latter, on account of their nearneſs, ſubtend greater angles. If then I have made no miſtake in tracing theſe matters, and if it cannot be eſtabliſhed by facts, that there is any time in our lives when the eye meaſures the apparent length and breadth of objects by the angles, which they ſubtend at it, there muſt be great reaſon, why we ſhould doubt, whether the eye ever does meaſure them or judge of them according to ſuch a rule or ſtandard; or whether we ſhould reckon the ſenſe of ſeeing to be anomalous or unlike the other four ſenſes, and unable, without experience, to diſcharge the duties allotted to it.

Hence then, ſince we do not learn the powers of diſtinguiſhing between different ſounds by our ears, or between different ſmells by our noſe, or between different taſtes by our palate, or between different feelings by our touch, we may believe, that we do not learn the power of diſtinguiſhing between the apparent magnitudes of different bodies or between the different apparent magnitudes of the ſame body by our eye-ſight, unleſs ſome fact can be brought which may be enough to over-rule ſuch a belief. Now I know of no fact, which can be alledged of that kind, but the recovery of couched eyes. But unſound or couched eyes have but an imperfect reſemblance or analogy to ſound eyes, or to the healthful eyes of children, and if they had a leſs imperfect reſemblance or analogy to them, they do not meaſure the apparent magnitude of objects by the real or geometrical angles, which ſuch objects ſubtend at the eye. For if a half-crown appears to a couched eye as big as a pewter-diſh, its apparent magnitude, as ſeen by ſuch an eye, cannot be meaſured by the ſame angle by which a geometrician wou'd meaſure it. Whence a child's eye, if like a couched eye, muſt alſo be unable to meaſure it by ſuch an angle. If then our eyes are not taught by experience like couched eyes, or if they are ſkilful as ſoon as we are born, like chickens eyes, it will be allowable to ſettle that they are accuſtomed to meaſure the apparent magnitude of all objects, near and diſtant, by the ſame rules, both in our childhood and at years of maturity.

PROPOSITION VI.

The ſun and moon ought to appear greater when ſeen in the horizon than when ſeen in the zenith.

Since the atmoſphere is a ſpherical refracting medium and more denſe than the medium, by which it is ſurrounded, the laſt images of the ſun and moon, when ſeen in the horizon, will, according to the firſt and ſecond and third propoſitions be

further

further off and greater, than when ſeen in the zenith, but will reſpectively ſubtend the ſame angle as well in one place as in the other. Yet ſince theſe laſt images are of ſuch a ſize, that they reſpectively ſubtend the ſame angle, altho' at different diſtances, thoſe of them, which are furtheſt from us, muſt, according to the corollary of the fifth propoſition, appear the greateſt, and for that reaſon the ſun and moon muſt appear greater, when ſeen in the horizon, than when ſeen in the zenith.

THE SECOND PART.

IT may be ſhewn, that the ſun and moon muſt appear greater in the horizon than in the zenith by another proof ſomewhat different from what I have already given.

EPERIMENT I.

I filled with water a glaſs-veſſel, which was four inches deep, and which was a ſegment of a hollow-ſphere, whoſe diameter was ſixteen inches; and afterwards I placed a candle under it nearer than the focus of parallel rays, and dipping my face into the water and looking at the candle I ſaw it erect and bigger than when ſeen with the naked eye in the open air. I then placed the candle further from this veſſel than the focus of parallel rays, at a diſtance of three or four yards, and looking at it, as before, it appeared no longer like a candle, but like two bright and circular planes, which reached over each other as far as to their centres, and of which each was of the ſize of a ſmall eating plate. I afterwards placed it at ten or twelve yards diſtance and found, that there were two bright and circular planes, as when it was placed at three or four yards diſtance, and that they were a little further increaſed.

EXPERIMENT II.

I filled alſo with water another glaſs-veſſel, which was eight inches deep, and which was half of a hollow-ſphere, whoſe diameter was ſixteen inches, and dipping my face into it at the place, which wou'd have been the centre of the whole veſſel, I looked at a candle placed under it at three or four yards diſtance and at a diſtance nearer than the focus of parallel rays, and notwithſtanding it appeared fainter in this than in the former experiment, yet it ſeemed as great or nearly as great at the ſame diſtances.

EXPERIMENT III.

I took a large pin and dipped the head of it into a ſmall ſpherical glaſs-veſſel full of water, and looked at it in air thro' the ſpherical ſurface of the veſſel, and it appeared ſenſibly increaſed.

Cor. 1. Hence if an object be dipped into water and ſeen in air thro' a ſurface, which is ſpherically concave, it will appear decreaſed.

Cor. 2. Hence on the contrary, if objects be placed in air and ſeen in water thro' a ſurface, which is ſpherically concave towards the water, they will appear increaſed, according to the account, which I have given in the firſt experiment, of objects placed and ſeen in ſuch ſituations.

Cor. 3. Hence the apparent magnitude of the heavenly bodies muſt be increaſed by the refraction of the atmoſphere; but ſuch refraction is not enough to counterbalance the decreaſe, which the apparent magnitude of thoſe bodies undergoes by their diſtance from us. For the denſity and on that account perhaps the refractive power of water is eight hundred times greater than the denſity or refractive power of the atmoſphere, and the apparent magnitude of objects is increaſed by the refraction of water in a much leſs proportion than of eight hundred to unity. Whence altho' the apparent magnitude of the heavenly bodies is ſomewhat increaſed by the refraction of the atmoſphere, yet ſuch increaſe is much leſs than the decreaſe, which they undergo by their diſtance from us.

PROPOSITION VII.

Objects can appear increaſed, when ſeen by refraction, altho' the angles ſubtended at the eye by their laſt images, from which the rays ſeem to come, are leſs than the angles ſubtended by the objects themſelves. (fig. 8.)

Let *LMN* be a ſpherical refracting medium, which is more denſe than the medium around it; and let *C* be the centre of it; and on the produced radius *CMwy* at a diſtance nearer than the focus of parallel rays let the line or object *Hw* be erected at right angles; and let the centre *C*, and the point *H*, which is the other extremity of the object *Hw* be joined by the produced radius *CH*; and let the ray *Hd* be in the ſame plane as the pro-

produced radii *CH* and *Cw*, and let it fall on the refracting ſurface *LMN* between them in ſome point *d*; and let be drawn the radiues *Cd*. Now the ray *Hd* after refraction will not go in the direction *Hdn*, but will be turned towards the radius *Cd* in the direction *dp* and will meet the produced radius *CMwy* not in the point *n* but in the point *p*. Let then the ray *dp* be produced backwards, and ſince the rays coming from the object *Hw* will be diverging after refraction, it will meet the produced radius *CHx* in ſome point *x*, further from the refracting medium *LMN* than the point *H*; and from the point *x* let fall the line or object *xy* perpendicular to the produced radius *CMwy*, and the line or object *xy* will be the laſt image of the object *HW*. But ſince the angle *xpy* ſubtended by the laſt image *xy* is leſs than the angle, which the object *Hw* wou'd ſubtend at the point *p*, one wou'd think, that ſuch object wou'd appear leſs by refraction than without refraction. Yet according to the firſt experiment it will appear greater by refraction. Whence the apparent magnitude of objects can be increaſed by refraction, altho' the angles ſubtended at the eye by their laſt images are leſs than the angles ſubtended by the objects themſelves.

I have hitherto ſuppoſed, that the eye of the ſpectator was placed in the axis of the pencil at ſome point *p* within the medium, and nearer to the object than the centre of the medium; I will now ſuppoſe, that it is placed in the point *C*, the centre of the medium, and then according to the ſecond experiment the object will appear increaſed by refraction as under the foregoing ſuppoſition. For altho' the two outer rays *HC* and *wC* will reach the eye without refraction, yet all the inner rays, which reach the eye between thoſe two rays, will be refracted and will ſeem to come as before, from ſome laſt image *xy* farther off than the object *Hw*. Whence, as in the former caſe, the object *Hw* will appear increaſed by refraction; and its laſt image, which is bigger than itſelf will ſubtend in this inſtance an equal angle. It is alſo further remarkable, that convex and concave lenſes ſeem to teach ſomewhat like what is aſſerted in this propoſition. For the former, which increaſe the apparent magnitude of objects always increaſe their laſt images, but the latter, which leſſen the apparent magnitude, always leſſen their laſt images, altho' in both of them the objects and laſt images ſubtend the ſame angles. (Smith's Opticks, Art. 255.)

If the object is placed in the focus of parallel rays or beyond the focus of parallel rays, the laſt image will be flung to an infinite diſtance, or beyond an infinite diſtance, and altho' the object will appear increaſed, yet the laſt image, if there is any laſt image in thoſe caſes, will ſubtend at the eye a leſs angle than the object itſelf.

AN ANALOGOUS CONSEQUENCE.

Hence ſince the atmoſphere is a ſpherical refracting medium like a ſpherical glaſs-veſſel of water, the ſun, the moon, the planets, and all other heavenly bodies muſt in all ſituations, both in the zenith and in the horizon and between thoſe places, appear increaſed by the former of them. It is true indeed, that the atmoſphere is much leſs convex and much leſs denſe than a ſpherical glaſs-veſſel of water, whoſe diameter is ſixteen inches; yet it has a certain degree of convexity and is more denſe, than the medium, by which it is ſurrounded, and ſince we are reckoned to be fifty miles deep in it, its refractive powers, be what they may, as far as they prevail, muſt work the ſame way and ſenſibly reſemble the refractions of a ſpherical glaſs-veſſel of the ſize, of which I have made mention. Sir Iſaac Newton has ſaid, according to his table of refractions, which Dr. Halley has given us in the Philoſophical Tranſactions, N°. 368. that the refraction of the atmoſphere in the horizon is ſo great, as cauſes a ray of light after refraction, if produced backwards, to make with the ſame ray of light before refraction an angle of thirty-three minutes and forty-five ſeconds. Whence as objects, ſeen in a ſegment, leſs than half of a ſpherical glaſs-veſſel of water, appear increaſed after refraction, altho' their laſt images ſubtend at the eye a leſs angle than the objects themſelves; ſo likewiſe, while the heavenly bodies appear increaſed by the refraction of the atmoſphere, their laſt images ſubtend at the eye leſs angles than the heavenly bodies themſelves. If they were ſeen at the centre of the earth, they wou'd ſubtend the ſame angles as their laſt images; and ſince a ſemidiameter of the earth bears no ſenſible or but an inconſiderable proportion to their reſpective diſtances from us, they may be ſaid, nearly with truth, to ſubtend the ſame angles as their laſt images on the ſurface of it. Beſides as the laſt images increaſe, while the objects are withdrawn from the ſegment of the glaſs-veſſel of water, and as the objects themſelves at the ſame time appear to increaſe; ſo likewiſe much rather muſt the heavenly bodies which, during the whole day, are always nearly at the ſame diſtance, appear the greateſt, in thoſe ſituations, where their laſt images are the greateſt and the furtheſt from us. Now the laſt images of thoſe bodies, according to the three firſt propoſitions are greateſt in the horizon and leaſt in the zenith. Whence the ſun and moon muſt appear greater when ſeen in one ſituation than in the other.

A RECAPITULATION.

I have ſhewn in the three firſt propoſitions of the firſt part, that the laſt images of the ſun and moon and other heavenly bodies are greater in the horizon than in the zenith, and that altho' they ſeem always raiſed by refraction, yet they may be or are ſometimes in reality lowered by it. I have ſhewn alſo in the fourth propoſition, that the angles, which objects, ſeen without refraction, ſubtend at the eye, increaſe ſomewhat faſter, than the diſtances of ſuch objects from us, decreaſe. I have ſhewn likewiſe in the fifth propoſition, that we do not form our judgement concerning the apparent magnitude of objects by the angles, which they ſubtend at the eye; and that greater and more diſtant objects can and do appear greater than leſs objects, altho' the former ſubtend at the eye but equal or leſs angles. I have ſhewn beſides in the ſixth propoſition, becauſe the laſt images of the ſun and moon and other heavenly bodies are greater in the horizon than in the zenith; and becauſe greater and more diſtant objects can and do appear greater than leſs objects, altho' the former ſubtend at the eye but equal or leſs angles, I ſay, I have ſhewn, that the ſun and moon and other heavenly bodies themſelves on theſe accounts muſt appear, as they always do appear, greater in the horizon than in the zenith.

In the ſecond part I have, by one experiment, ſhewn, contrary to what has uſually been thought to be the truth, that when an object, placed in air is viewed in a ſegment leſs than half of a ſpherical glaſs-veſſel of water, it will appear increaſed, altho' the angle ſubtended at the eye by its laſt image is leſs than the angle ſubtended by the object itſelf. By another experiment I have ſhewn, that objects placed in air appear alſo increaſed, when they are viewed from the centre of a ſpherical glaſs-veſſel of water, altho' all opticians agree, and teach, that objects thus placed and ſeen, will appear neither increaſed nor leſſened. I have ſhewn too by both theſe experiments, that if theſe objects are removed further off, their laſt images will become greater, yet will ſubtend at the eye leſs angles, and that their apparent magnitudes notwithſtanding will be more increaſed. Hence then I have analogically concluded, that the laſt images and apparent magnitudes of the ſun and moon and other heavenly bodies muſt be increaſed by the refraction of the atmoſphere; and ſince their laſt images, according to the three firſt propoſitions muſt be greateſt in the horizon that their apparent magnitudes muſt alſo be the greateſt in that ſituation.

PROPOSITION VIII. (fig. 9.)

The ſuperior planets can have a retrograde motion in their oppoſition, altho' the ſun moves in an orbit round the earth.

It is poſſible, that the orbit of a ſuperior planet may be ſo ſmall, and the motion in its orbit ſo great, that it ſhall always be progreſſive. Thus if the motion in its orbit is grearer than the greateſt increaſe or loſs, which it ever receives in its motion, from the motion of its orbit, it never can be retrograde. For the increaſe or loſs, which it receives in its motion from the motion of its orbit is never equable; but is reſpectively retarded or accelerated as it approaches towards its oppoſition, and reſpectively accelerated and retarded as it retires from it. Yet the orbit of a ſuperior planet may be ſo great, and its motion in that orbit ſo much leſs, than the increaſe or loſs, which it receives in its motion from the motion of its orbit, as will render it retrograde, while it is approaching towards its oppoſition and paſſing beyond it. For let the orbit of a ſuperior planet be much greater than the ſun's orbit, and let the motion, which it receives from the motion of its orbit along with the ſun, be much greater than the motion, which it has in its own orbit, and let *E* be the earth, and let the circle *STUV*, drawn round the centre *E*, be the ſun's orbit; and let *S* be the place, where the ſun is in its orbit, and let the circle *POQ*, drawn round the centre *S*, be the orbit of a ſuperior planet; and let the ſuperior planet *P* be in its conjunction, and let the circle *ABC*, drawn round the centre *E* reach to what are called the fixed ſtars; then the planet *P* will ſeem near the ſtar *b*. Now if the ſun ſhou'd move from the point *S* thro' a quarter of its orbit to the point *T*, the ſuperior planet *P* will move, by the motion of its orbit to the point *R* in the circle drawn round the centre *T*, and by the motion in its orbit to ſome point 1*p* in the ſame circle. Whence ſince each of theſe motions are progreſſive, the whole motion compounded of both of them, will alſo be progreſſive. Now if the ſun ſhou'd move from the point *T* thro' another quarter of its orbit to the point *U*, the ſuperior planet at the point *R* will move by the motion of its orbit to the point *L* in the circle, drawn round the centre *U*, and by the motion in its orbit to ſome point 2*p* in the ſame circle, making the arc *L*2*p* equal to twice the arc *R*1*p*, and will be progreſſive, becauſe both theſe motions are progreſſive, and will be ſeen near the ſtar *C*. Now if the ſun ſhou'd move from the point *U* thro' another quarter of its orbit to the point *V*, the ſuperior planet at the point *L* will move, by the motion of its orbit to the point *M* in the circle drawn round the centre *V*, and by the motion in its orbit, to ſome point 3*p* in the ſame circle, making the arc *M*3*p* three times as great as the arc *R*1*p*. If then the motion of this planet is ſo ſlow, as to make it paſs thro' its oppoſition, while the ſun moves from the point *U* to the point *V*, the motion, during that time, and before and after that time, which it receives from the motion of its orbit, will be contrary to the motion, which it has in its own orbit; and becauſe it is greater, according to the ſuppoſition, the motion, which is compounded of both of them or which is the difference of both of them, muſt be retrograde. Whence this planet at the point 3*p*, will be ſeen at the ſtar *B*, which is nearer the point *b*, from which it begun its motion, than the point *C*; and will have moved backwards thro' a ſpace equal to *CB*, while the ſun moved thro' the quarter of a circle *UV*.—It may be ſhewn likewiſe after the ſame manner, that this ſuperior planet in all other places of its orbit, will have a retrograde motion, whenever it paſſes thro' its oppoſition. Whence ſuperior planets may have a retrograde motion, altho' the ſun moves in an orbit round the earth.

Since what are called the fixed ſtars have no annual motion, they render it likely, that the ſun moves in an orbit round the earth, and that the earth does not move in an orbit round the ſun.

ASTRONOMICAL DIFFICULTIES.

I.

Modern aſtronomers hold, that the earth's ſurface at the equator, on account of its duirnal rotation, moves according to a rate of ſixteen or ſeventeen miles a minute, and that the earth itſelf moves in its orbit according to a rate of a thouſand miles a minute. Now ſince the ſloweſt of theſe motions is perhaps as ſwift as lightning and ſince the earth ſeems to ſtand quite ſtill, I cannot, for my own part, help entertaining doubts, whether the earth's ſurface or the earth itſelf have ſuch motions.

II.

If we look at a ſteeple or building or hill at a diſtance, which is equal to a hundred and fifteen times its height, it will ſeem as high or nearly as high, as if we ſtood near to it. Yet the ſun, whoſe diameter is eight hundred and fifty-four thouſand miles, and whoſe diſtance is not more than a hundred and fifteen times its diameter, and whoſe apparent magnitude is increaſed by the refraction of the atmoſephere, appears no bigger than a Cheſhire cheeſe. It is then ſubmitted, how far we have room to doubt concerning what is ſaid to be the ſun's ſize and the ſun's diſtance.

III.

Since the ſun in the ſolſtitial points, on account of the obliquity of the ecliptic, is five millions of miles further from us or thereabouts than in the equinoctial points, we may reaſonably aſk, how it happens, that he does not ſeem proportionally leſs in one ſituation than in the other.

IV.

Since the moon, at its greateſt latitude from the equator, is five or ſix ſemidiamters of the earth further from us than in the equator, how does it happen, that ſhe does not appear proportionally leſs in one ſituation than in the other.

V.

If the earth moves round the ſun, its motion is compounded of a motion, with which it was projected in a ſtraight line, and of another motion, which ariſes from the attraction of the ſun. But if the earth has a diurnal rotation round its own axis, ſuch diurnal rotation is unbiaſſed by the ſun's attraction, and uncompounded with any motion on either ſide of the equator, which can ariſe from it. For ſince the ſun is ſometimes on the north ſide and ſometimes on the ſouth ſide of the equator, his attraction wou'd ſway the earth's diurnal rotation, if it did ſway it, one while on one ſide and one while on the other. But ſince nothing of this ſort ever happens, we muſt believe either that the earth has no diurnal rotation, or that the ſun has no influence on it. If then the earth has a diurnal rotation, what is the reaſon, why the ſun, which is reckoned able to draw the earth from its projectile direction, is not alſo able at the ſolſtices, when it acts obliquely on ſuch diurnal rotation, to draw it from its direction in the equator. It is indeed allowed, that if the earth always moved in the plane of the equator, its axis, notwithſtanding the ſun's attraction, wou'd continue to be parallel to itſelf. But at the ſummer ſolſtice, the ſun is ſaid to be twenty or thirty millions of miles from the plane of the equator, and acts with leſs obliquity on the diurnal rotation of the northern parts of the earth than on the diurnal rotation of the ſouthern parts of the earth; whence it muſt much more biaſs the northern parts and draw the north-pole towards it. Doctor Keil ſays (page 66.) that if the earth moved in its orbit without a diurnal rotation, any one of its diameters, however inclined to the ecliptic, wou'd always move parallel to itſelf, and thence infers, that if the earth ſhou'd be impreſſed with a diurnal rotation round ſuch diameter, ſuch diameter, wou'd ſtill keep its paralleliſm. Now I grant, that if the earth moved in its orbit without a diurnal rotation, all its diameters wou'd keep parallel to themſelves, becauſe the earth's ſurface wou'd have no motion acroſs the ecliptic. But if the earth's ſurface has a motion acroſs the ecliptic, in the direction of the equator, I then hold that the attraction of the ſun, for the reaſons already given, muſt cauſe the earth's axis to loſe its paralleliſm.

VI.

If the ſun ſhou'd be in the tropic of cancer, all the earth's meridians of longitude thro' the whole ſurface of attraction on the ſouth-ſide of ſuch tropic wou'd move one way with reſpect to the ſun; but the meridians of longitude on the north-ſide wou'd move the ſame way thro' only ſixty-ſix degrees and a half of the ſurface of attraction, and thro' the remaining twenty-three degrees and a half wou'd move a contrary way with reſpect to the ſun, and partly in the direction of the ſun's attraction. Whence their contrary motion thro' theſe twenty-three degrees and a half muſt aſſiſt in deſtroying the paralleliſm of the earth's axis and muſt give a further biaſs to the north-pole, beſides that, of which I have ſpoken, as ariſing from the obliquity of the ecliptic and the equator.

VII.

If the earth and the atmoſphere were impreſſed with a diurnal motion round the earth's axis, and met with no external interruption, they wou'd both alike go for ever according to ſuch impreſſion. But the ſun and moon by the influence of heat and attraction wou'd ſoon retard the motion of the atmoſphere, and if the influence of the ſun and moon ſhou'd ceaſe, after they had cauſed a certain degree of retardation, ſuch retardation wou'd not ceaſe likewiſe, but wou'd ſtill continue, and the atmoſphere wou'd move ſlower than the earth. If likewiſe after ſome time the influence of the ſun and moon ſhou'd return, then ſuch retardation wou'd further increaſe, till the whole of the atmoſphere's motion wou'd be deſtroyed, or till it wou'd ſeem to move toward the weſt, as faſt as the ſurface of the earth moved toward the eaſt. Whence there wou'd always be between the tropics an eaſt-wind, which wou'd move ſixteen or ſeventeen miles an hour altho' there are no winds of which we have any knowledge, that move faſter than one mile an hour.

hour. Whence the earth and atmoſphere cannot have been impreſſed with any diurnal rotation from weſt to eaſt; neither can it be rightly ſaid, that they have received a diurnal rotation of this ſort from the attraction of the ſun and moon, and that the heat of the ſun cauſes a ſlight retardation of ſuch diurnal rotation of the atmoſphere and makes conſtantly, as between the tropics, an eaſterly wind. Becauſe ſuch an attraction wou'd move the ſurface of the earth, not in the direction of the equator, in which it now moves, but in a direction, which wou'd be between the ecliptic and the moon's orbit.

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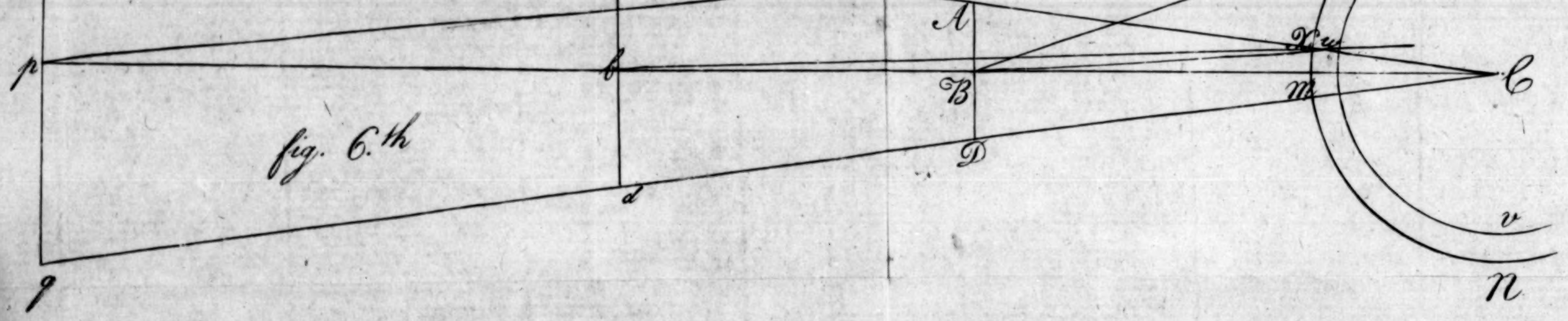
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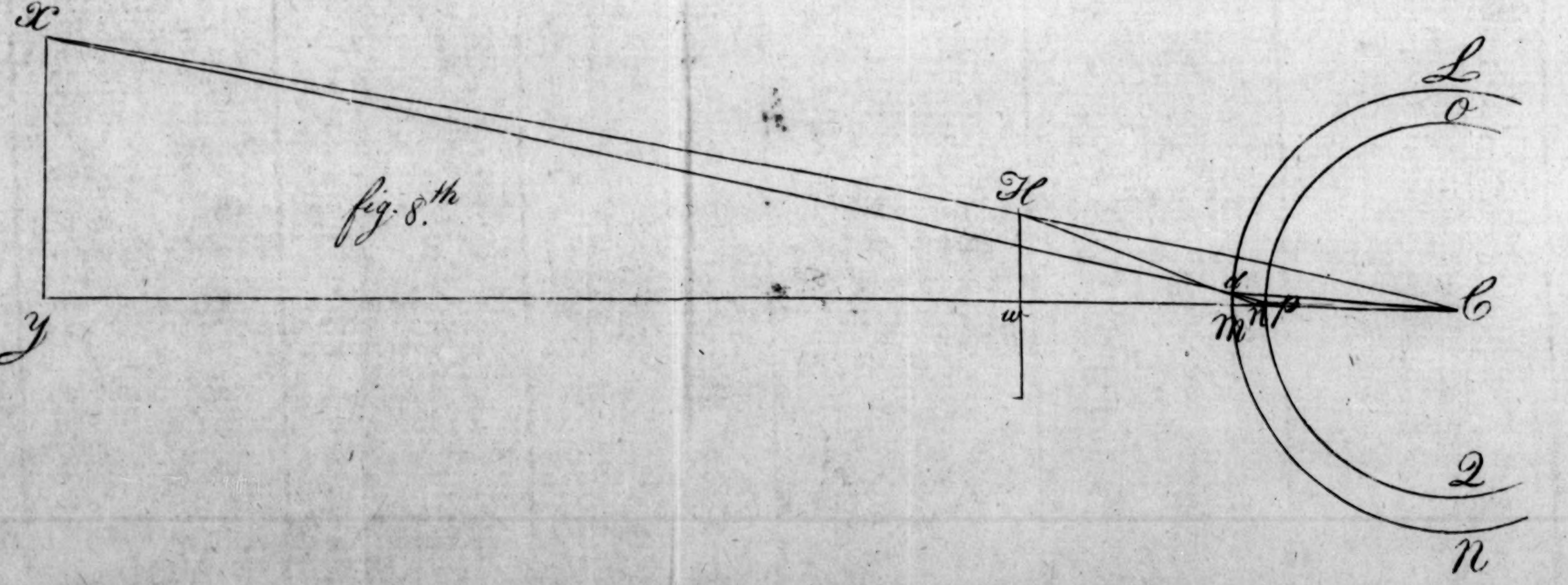
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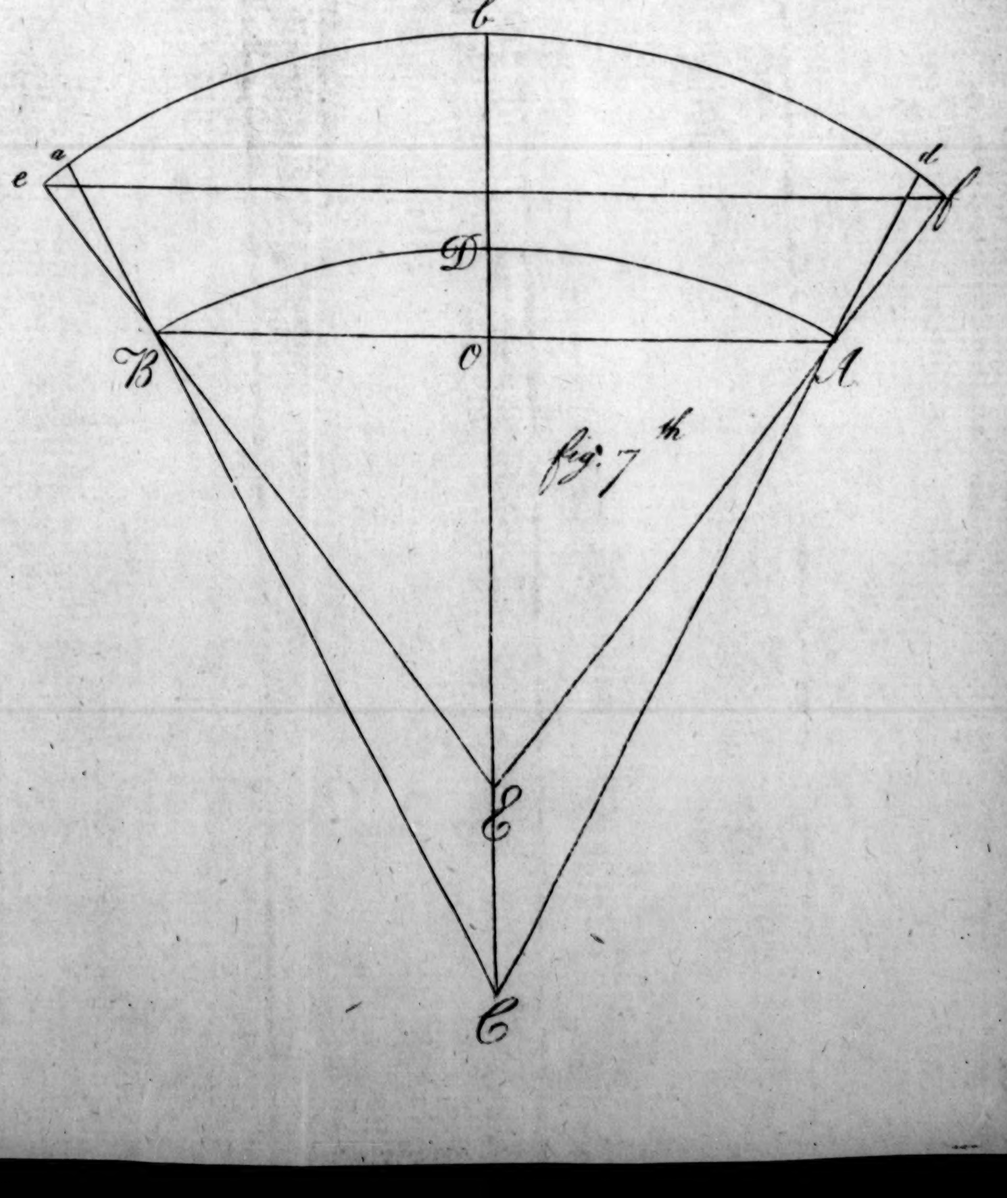
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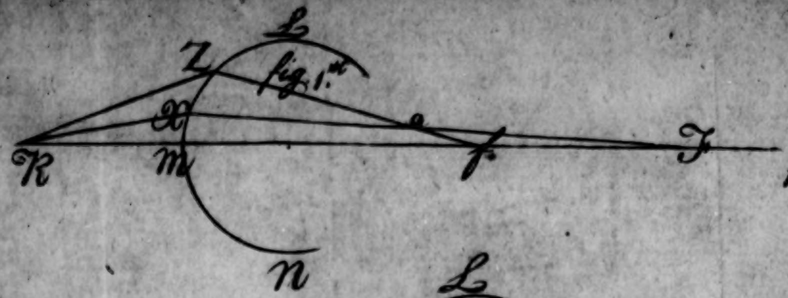
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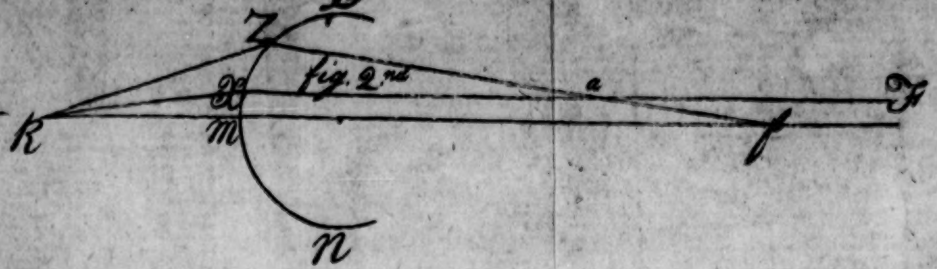
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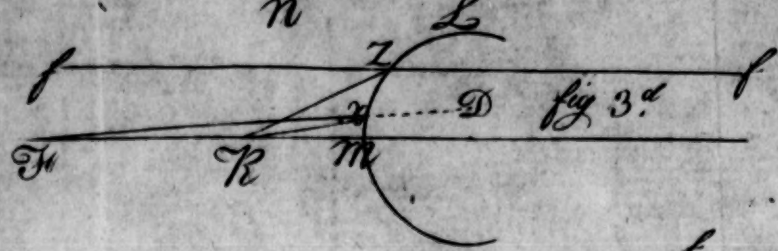
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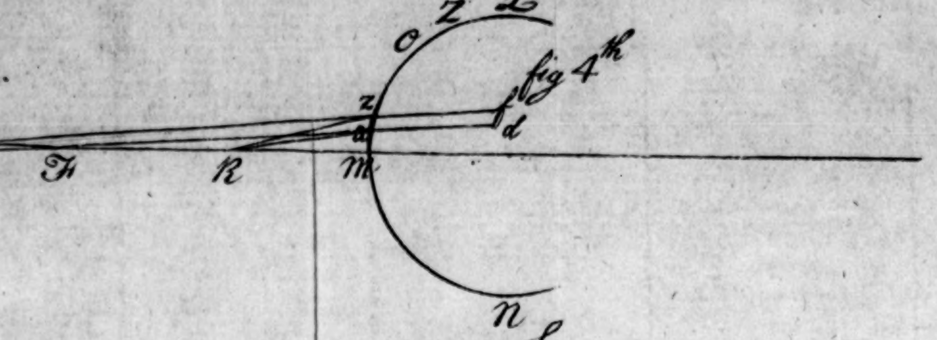
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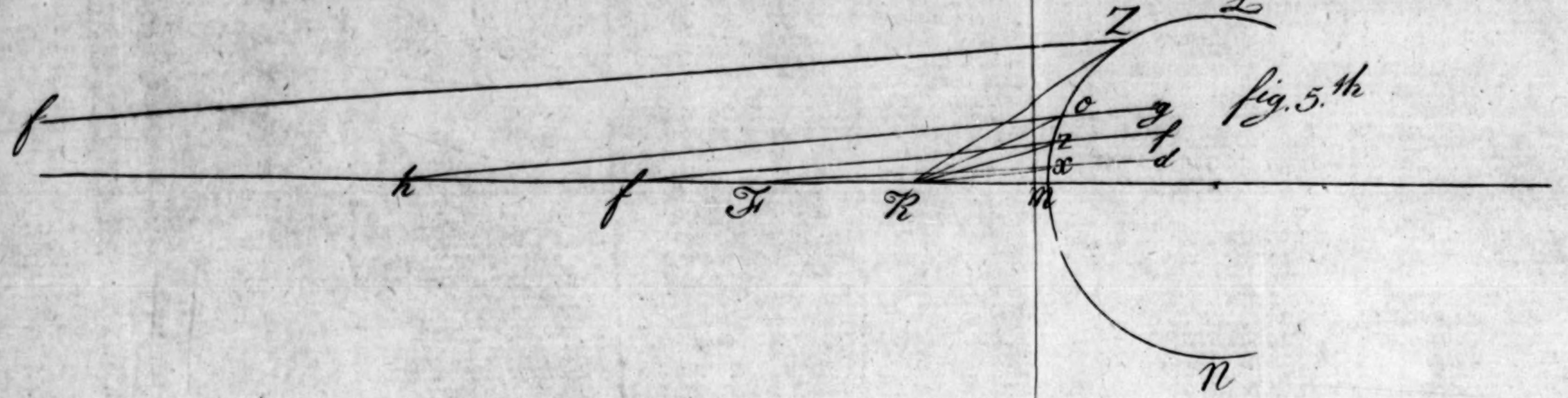
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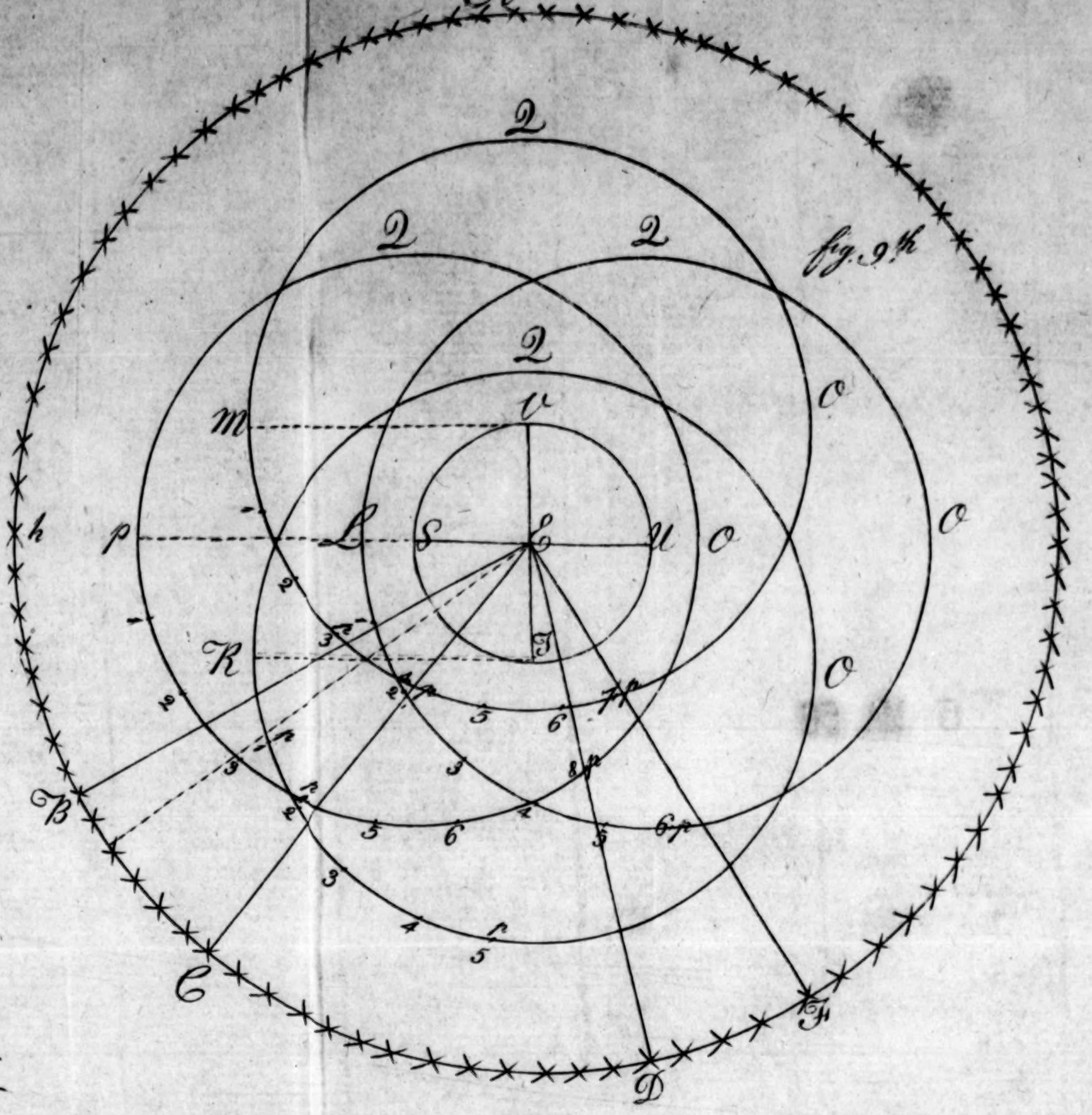
fig. 9th